AERO MATHEMATICS CURRICULUM FRAMEWORK
HIGH SCHOOL STANDARDS
Adopted from the Common Core Standards

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Introduction

AERO CURRICULUM FRAMEWORK IN MATHEMATICS (9-12)

Introduction

When curriculum standards were first developed in the 1990's, they were typically organized into grade spans. The goal was to allow curriculum flexibility with the idea that a student was expected to understand the concept, "By the End of Grade…" The general guide for the grade placement of standards within grade bands was perceived beliefs within the content community about when and how the big ideas of a discipline unfolded. Standards were in turn used by educators to develop aligned curriculum frameworks and later, accountability systems.

In light of current information being derived from cognitive research, the sequential arrangement of topics and concepts that are embedded in broad standards is being rethought. These ideas are now being mapped into learning progressions, webs of interconnected ideas that illustrate how over time, student's ideas deepen and become more sophisticated and do so in predictable ways. Research-based learning progressions enable us to reframe how we think about Standards and Learning Expectations for students and to view them from a long-range perspective. Similar progressive changes in what people are capable of doing are also being plotted.

The Way It Works…

The idea of using information drawn from research-based learning progressions as a framework for supporting curriculum reform is rapidly gaining acceptance. A student's knowledge base expands as they become increasingly able to make logical cognitive connections between different but related concepts. By cognitive connections we mean relationships among ideas in which knowledge of one concept contributes to understanding other larger ideas. Within a learning progression, clusters of interrelated ideas become increasingly sophisticated across grade levels and subject areas. The metaphor of a ladder of understanding illustrates why the mastery of precursor ideas and skills introduced in earlier courses and grade levels is essential for making the type of cognitive leaps that results in a deep and flexible understanding of the central concepts within a discipline.

Applying novice to expert learning progressions as a framework for organizing the K-12 curriculum is consistent with research showing that deep and lasting learning occurs best:
1. over time,
2. through repeated and scaffolded exposures to concepts, and
3. when encountered in a variety of educational contexts.

The AERO Mathematics Curriculum Framework Grade and Course Level Expectations outline related ideas, concepts, skills and procedures that form the foundation for understanding and learning mathematics. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated in the STEM standards. The standards provide a framework to bring focus to teaching, learning, and assessing mathematics.

The Grade Level Expectations (GLEs) in grades K-8 specify mathematical content that students need to understand deeply and thoroughly for future mathematics learning. At the high school level, the set of core standards and performance descriptors for each for the five content strands are presented. These learning progressions outline the mathematics expectations for all students include STEM courses. The indicators have been identified as core to mathematics at these levels. See Appendix A to see how these progressions constitute the traditional high school Algebra, Geometry, and Algebra 2 courses. The common core website (www.commoncore.org) lists the content for Integrated math 1, 2, and 3.

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated in the STEM standards. All graduates are expected to achieve the core standards. students **Advanced (STEM) standards are not intended to apply to all high school.** Advanced standards are intended to apply to students having achieved the core mathematics standards and are more advanced than first and second year algebra and basic geometry. Not every student will choose to enroll in advanced mathematics curriculum; some students do not choose to take more than core mathematics requirements. Students who plan to attend post-secondary educational institutions should complete mathematics courses reflected in the advanced standards. Taking rigorous mathematics courses will be important to success in post-secondary educational studies.

The high school standards are listed in conceptual categories: These conceptual categories continue the progressions identified in the K-8 Curriculum framework.

· Number and Quantity
· Algebra and Functions
· Modeling
· Geometry
· Statistics and Probability.

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, *why* a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities reading should allow for use of Braille, screen reader technology, or other assistive devices, while writing should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students

Essential content included in the Grade and Course Level Expectations should be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations. These are identified in the mathematical processes standards.

NOTE: There is a difference between the intent of the AERO Mathematics Curriculum Framework and the Common Core. The Common Core was developed from the standpoint of instruction and not only includes the math required of all students but also describes additional mathematics that students should learn to pursue careers and majors in science, technology, engineering and mathematics **(STEM) fields indicated by a + sign**. The AERO Mathematics Framework was developed as a document to inform assessments. The AERO Mathematics Curriculum Framework has been adapted from the Common Core.

Depth-of-Knowledge (DOK) Levels for Mathematics

Each indicator for the AERO Mathematics Curriculum Framework has been assigned a Depth of Knowledge (DOK) level based on the work of Norman L. Webb, Wisconsin Center for Educational Research ("Depth-of-Knowledge Levels for Four Content Areas," March 28, 2002), DOK levels measure the degree to which the knowledge elicited from students on assessments are as complex as what students are expected to know and do as stated in the performance indicators. According to Webb, "interpreting and assigning depth-of-knowledge levels to both objectives within standards and assessment items is an essential requirement of alignment analysis. Instruction, assignments, and classroom assessment must incorporate the same expectations. DOK levels for an indicator must mirror the DOK level for the assessment.

DOK levels help administrators, teachers, and parents understand the intent of the indicators, in terms of the complexity of what students are expected to know and do. Indicators vary in terms of complexity. Some expect students to reproduce a fact or complete a sequence of steps, while others expect students to reason, extend their thinking, synthesize information from multiple sources, and produce significant work over time. Teachers must know what level of complexity is required by an indicator in order to ensure that students have received prior instruction or have had an opportunity to learn content at the level students will be expected to demonstrate or perform. Assessment items must be created to ensure that what is elicited from students on the assessment is as demanding cognitively as what students are expected to know and do as stated in the indicators.

Four levels of Depth of Knowledge (DOK) are used in the AERO Mathematics Curriculum Framework.

DOK 1 (Recall) includes the recall of information such as fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include "identify," "recall," "recognize," "use," and "measure." Verbs such as "describe" and "explain" could be classified at different levels depending on what is to be described and explained.

DOK 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify," "organize," "estimate," "make observations," "collect and display data," and "compare data." These actions imply more than one step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Some action verbs, such as "explain," "describe," or "interpret" could be classified at different levels depending on the object of the action. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

DOK 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

DOK 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections-relate ideas within the content area or among content areas-and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level.

Mathematical Processes

The Common Core Standards identify eight Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

These practices describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important **"processes and proficiencies"** with longstanding importance in mathematics education.

The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. They are identified as the first four standards in the AERO Framework for Mathematics.

The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1.0 Problem Solving

Students will:

- Recognize and use mathematical ideas and processes that arise in different settings, with an emphasis on formulating a problem in mathematical terms, interpreting the solutions, mathematical ideas, and communication of solution strategies.

- Apply and adapt a variety of appropriate strategies to problem solving, including testing cases, estimation, and then checking induced errors and the reasonableness of the solution.

2.0 Reasoning and Proof

Students will:

- Develop inductive and deductive reasoning to independently make and evaluate mathematical arguments and construct appropriate proofs; include various types of reasoning, logic, and intuition.

3.0 Communication

Students will:

- Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely in mathematical reasoning.

- Employ reading and writing to recognize the major themes of mathematical processes, the historical development of mathematics, and the connections between mathematics and the real world.

4.0 Connections

Students will:

- Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic), to solve problems, to model mathematical ideas, and to communicate solution strategies.

Numbers and Number Systems.

Introduction

During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers.

During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers.

In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers .With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. Extending the properties of whole-number exponents leads to new and productive notation. Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

5.0 Numbers and Operations
Students will understand and apply numbers, ways of representing numbers, relationships among numbers, and number systems.

Number and Quantity: The Real Number System Domain	**Students will understand:** How properties of rational exponents, rational number, and irrational number are defined using characteristic patterns of equivalency and computation, to build a comprehensive knowledge of the structure and order of the real number system. **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will:	**5.1 Extend the properties of exponents to rational exponents**			
Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number	5.1.1.A Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. F*or example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* *DOK 1*	5.1.2.A Rewrite expressions involving radicals and rational exponents using the properties of exponents. *DOK 1*	5.1.3.A Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. *DOK 1*	**STEM** Select and use an appropriate form of number (integer, fraction, decimal, ratio, percent, exponential, scientific notation, irrational, complex) to solve practical problems involving order, magnitude, measures, labels, locations and scales. *DOK 2*
	5.1.1.B Compare and order numbers in the Real Number System by size and/or position on a number line (to include ability to identify equivalent terms). *DOK 2*	5.1.2.B NA	5.1.3.B NA	Judge the effects of computations with powers and roots on the magnitude of results. *DOK 2*

By the end of Grade 8, students will:	**5.2 Use properties of rational and irrational numbers.**			
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).	5.2.1.A Explain why the sum or product of two rational numbers is rational; *DOK 3*	5.2.2. A Explain that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational; *DOK 3*	5.2.3.A Justify mathematical procedures and determine how they apply to invented operations using field properties (closure, associative, commutative, distributive, identity and inverse). *DOK 3*	**Stem** NA
Mathematical Practice:	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Equivalency, Real Number System, Rational Numbers, Irrational Numbers, Integers, Radical.** **See Standards for Mathematical Practice: Appendix B**			
Complex Numbers	**Students will understand:** How knowledge of number properties in the Real Number System can be use to develop and apply properties of the Complex Number System **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will:	**5.3 Perform arithmetic operations with complex numbers.**			
NA	5.3.1.A Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real. *DOK 1*	5.3.2.A Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply	5.3.3.A Solve quadratic equations with real coefficients that have complex solutions.	**Stem** (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x -$

	.	complex numbers *DOK 1*		*2i). DOK 1*
By the end of Grade 8, students will:	**5.4 Represent complex numbers and their operations on the complex plane.**			
NA	**NA**	**NA**	**NA**	**Stem** **(+)** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. *DOK 1*
	NA	**NA**	**NA**	**Stem** **(+)** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120°.* *DOK 1*
	NA	**NA**	**NA**	**Stem** **(+)** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its

				endpoints. *DOK 2*
	NA	**NA**	**NA**	**Stem** **(+)** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex number . *DOK 2*
By the end of Grade 8, students will:	**Continued** **5.4 Represent complex numbers and their operations on the complex plane.**			
NA	**NA**	**NA**	**NA**	**Stem** (+) Know the Fundamental Theorem of Algebra show that it is true for quadratic polynomials. *DOK 2*
Mathematical Practice:	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Complex Number, Real Number, Quadratic Function.** **See Standards for Mathematical Practices Appendix B**			

Vectors and Matrices	**Students will understand:** How knowledge of number properties in the Real Number System can be use to develop and apply properties of vectors and matrices **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will:	**1.5** **Represent and model with vector quantities.**			
NA	NA	NA	NA	**STEM** (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v). DOK 1
NA	NA	NA	NA	**STEM** (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. DOK 1
NA	NA	NA	NA	**STEM** (+) Solve problems involving velocity and other quantities that can be represented by vectors. DOK 2

By the end of Grade 8, students will:	**1.6** **Perform operations on vectors.**			
NA	NA	NA	NA	**STEM** (+) Add and subtract vectors. DOK 1
NA	NA	NA	NA	**STEM** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. DOK 2
NA	NA	NA	NA	**STEM** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. DOK 1
NA	NA	NA	NA	**STEM** Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. DOK 2

By the end of Grade 8, students will:	**1.6 continued** **Perform operations on vectors.**			
NA	NA	NA	NA	**STEM** (+) Multiply a vector by a scalar. DOK 1
NA	NA	NA	NA	**STEM** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. DOK 2
NA	NA	NA	NA	**STEM** Compute the magnitude of a scalar multiple *cv* using $\|\|cv\|\| = \|c\|v$. Compute the direction of *cv* knowing that when $\|c\|v \neq 0$, the direction of *cv* is either along *v* (for $c > 0$) or against *v* (for $c < 0$). DOK 1

By the end of Grade 8, students will:	**1.7** **Perform operations on matrices and use matrices in applications.**			
NA	NA	NA	NA	**STEM** (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. DOK 2
NA	NA	NA	NA	**STEM** (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. DOK 1
NA	NA	NA	NA	**STEM** (+) Add, subtract, and multiply matrices of appropriate dimensions. DOK 1
NA	NA	NA	NA	**STEM** (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. DOK 2

By the end of Grade 8, students will:	**1.7 Continued** **Perform operations on matrices and use matrices in applications.**			
NA	NA	NA	NA	**STEM** (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. DOK 1
NA	NA	NA	NA	**STEM** (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. DOK 2
NA	NA	NA	NA	**STEM** (+) Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area. DOK 2

Quantities

Introduction

In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume.

In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification.

Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

6.0 MEASUREMENT **Students will use concepts and tools of measurement to describe and quantify the world. Students will be able to:**				
Number and Quantity: Quantities Domain	**Students will understand:** How unit and scale can be used as a tool to effectively model context and solve problems. **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will:	**6.1 Reason quantitatively and use units to solve problems**			
Estimate and convert units of measure for mass and capacity within the same measurement system (metric).	6.1.1.A Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. DOK 1	6.1.2.A Define appropriate quantities for the purpose of descriptive modeling. DOK 1	6.1.3.A Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. DOK 1	**Stem** NA
Mathematical Practice:	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **scale, quantity, accuracy** See Standards for Mathematical Practices Appendix B			

ALGEBRA

Introduction

Expressions

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b1+b2)/2)h$, can be solved for h using the same deductive process. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

7.0 Patterns, Functions, and Algebra
Students will use various algebraic methods to analyze, illustrate, extend, and create numerous representations (words, numbers, tables, and graphs) of patterns, functions, and algebraic relations as modeled in practical situations to solve problems, communicate, reason, and make connections within and beyond the field of mathematics.

Seeing Structure in Expressions	**Students will understand that:** Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will:	**7.1 Interpret and model a given context using expressions**			
. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	7.1.1.A Interpret expressions that represent a quantity in terms of its context. DOK 1	7.1.2.A Interpret parts of an expression, such as terms, factors, and coefficients. DOK 1	7.1.3.A Interpret complicated expressions by viewing one or more of their parts as a single entity. DOK 1	**Stem** NA
	7.1.1.B Use the structure of an expression to identify ways to rewrite it. DOK 1	7.1.2.B NA	7.1.3.B NA	**Stem** NA

By the end of Grade 8, students will:	**7.2 Decompose and recompose algebraic expressions using number properties in the context of solving problems**			
Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	7.2.1.C Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* DOK 2	7.2.2.C Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. DOK 1	7.2.3.C Use the properties of exponents to transform expressions for exponential functions. DOK 1	**Stem** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. DOK 3
Mathematical Practice:	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Expression, Term, Coefficient, Geometric Series, Exponential Function, Quadratic Function, Complete the Square, Factor, Zeros, Roots, Intercept, Maximum, Minimum.** **See Standards for Mathematical Practices Appendix B**			

Arithmetic with Polynomials and Rational Expressions	**Students will understand:** How to extend and apply the conceptual understanding of arithmetic structures and operation to polynomials. Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will	**7.3 Demonstrate that polynomials form a system analogous to the integers, namely, they are closed under the operation of addition, subtraction, and multiplication.**			
The conceptual understanding of arithmetic structures and operations is achieved by grade 8 and now extending this to polynomials	7.3.1.A Add, subtract, and multiply polynomials. DOK 1	7.3.2.A Prove polynomial identities and use them to describe numerical relationships. DOK 2	7.3.3.A Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. DOK 1	**Stem** (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. DOK 1

By the end of Grade 8, students will	**7.4 Demonstrate that polynomials can be decomposed and recomposed.**			
The conceptual understanding of arithmetic structures and operations is achieved by grade 8 and now extending this to polynomials	7.4.1.A Identify zeros of polynomials when suitable factorizations are defined by the polynomial. DOK 1	7.4.2.A Apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x-a is p(a), so p(a) = 0 if and only if (x-a) is a factor of p(x). DOK 1	7.4.3.A NA	**Stem** (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. DOK 2
Mathematical Practice:	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: Polynomial, Factor, Identity, Rational, Expression, Degree, Zeros, Roots See Standards for Mathematical Practices Appendix B			

Algebra: Creating Equations	**Students will understand that:** An equation is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Numeric relationships can be symbolically represented as equations and inequalities and fluency in transforming these symbolic representations is a tool for graphing and solving problems. **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will	**7.5 Demonstrate that the relationship of two or more variables can be represented as an equation or inequality and can be represented graphically**			
Solve linear equations in one variable	7.5.1.A Create equations and inequalities in one variable and use them to solve problems. DOK 2	7.5.2.A Create equations in two or more variables to represent relationships between quantities: graph equations on coordinate axes with labels and scales. DOK 2	7.5.3.A Represent constraints by equations or inequalities, and by systems and/or inequalities. DOK 2	**Stem** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. DOK 1
Mathematical Practice:	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Equation, Variable, Constraint, Linear, Quadratic, Rational, Exponential, Inequality, System of Equations** **See Standards of Mathematical Practice Appendix B**			

Algebra – Reasoning with Equations and Inequalities	**Students will understand that:** Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will	**7.6 Use algebraic properties and inverse operations to justify the steps in solving equations**			
Solve linear equations in one variable. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions.	7.6.1.A Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. DOK 3	7.6.2.A Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. DOK 1	7.6.3.A Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. DOK 1	**Stem** NA
By the end of Grade 8, students will	**7.7. Solve quadratic equations in one variable.**			
NA	7.7.1.A Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x-p)2 = q that has the same solutions Derive the quadratic formula from this form. DOK 2	7.7.2.A Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. DOK 1	7.7.3.A Recognize when the quadratic formula gives complex solutions and write them as a+- bi for real numbers a and b. DOK 1	**Stem** NA

By the end of Grade 8, students will	**7.8 Solve systems of equations**			
Analyze and solve pairs of simultaneous linear equations.	7.8.1.A Solve systems of linear equations exactly and approximately, focusing on pairs of linear equations in two variables. DOK 1	7.8.2.A Solve a simple system consisting of a linear equation and quadratic equation in two variables algebraically and graphically. DOK 1	7.8.3.A Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. DOK 3	**Stem** (+) Represent a system of linear equations as a single matrix equation in a vector variable. DOK 1
	NA	NA	NA	**Stem** (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater) DOK 1
By the end of Grade 8, students will	**7.9 Knows that a graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).**			
NA	7.9.1A Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y = g(x) intersect are the solutions of the equation f(x) =g(x); find the solutions approximately. DOK 2	7.9.2A Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. DOK 1	7.9.3.A NA	**Stem** NA

Mathematical Practice:	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Quadratic Formula, Complete the Square, Rational Equations, Radical Equations, Complex Number** See Standards for Mathematical Practice Appendix B

Functions

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions

Interpreting Functions Domain	Students will" Understand how the concept of function can be used to interpret, analyze and model functions that emerge from contexts including those contexts that are purely mathematical **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will	7.10. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).			
Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output	7.10.1.A Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. DOK 1	7.10.2.A Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1. DOK 2	7.10.3.A For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* DOK 2	**Stem** NA

By the end of Grade 8, students will	7.11 Describe characteristics of graphs, tables, and equations that model families of functions.			
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	7.11.1.A Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. DOK 2	7.11.2.A Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* DOK 2	7.11.3.A NA	**Stem** NA
Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear)	7.11.1.B Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. DOK 1	7.11.2.B Graph linear and quadratic functions and show intercepts, maxima, and minima. DOK 1	7.11.3.B Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. DOK 1	**Stem** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. DOK 1
Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph.	7.11.1.C NA	7.11.2.C Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. DOK 2	7.11.3.C Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude DOK 1	**Stem** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* DOK 2

By the end of Grade 8, students will	**7.12 Use strategies for interpreting key features of representations.**			
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	7.12.1.A Use the properties of exponents to interpret expressions for exponential functions. DOK 2	7.12.2.A Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function DOK 2	7.12.3.A Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum DOK 2	**Stem** NA
Mathematical Practices	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Domain, Range, Function Notation, sequence, symmetry, periodicity, intercept, zeros, maxima, minima, factorization, midline, amplitude,** *Function Family Names:* **Linear, Exponential, Quadratic, Trigonometric, Logarithmic, Polynomial, Square Root, Cube Root, Piecewise, Step Function, Absolute Value.** **See Standards for Mathematical Practices Appendix B**			

Functions - Building Functions Domain	**Students will understand that:** Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will	**7.13 Know the difference between a recursive rule and an explicit expression for a function.**			
NA	7.13.1.A Write a function that describes a relationship between two quantities.* DOK 2	7.13.2.A Find inverse functions. DOK 1	7.13.3.A Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. DOK 1	**Stem** (+) Compose functions. DOK 1
	7.13.1.B Determine an explicit expression, a recursive process, or steps for calculation from a context. DOK 2	7.13.2.B NA	7.13.3.B NA	**Stem** (+) Verify by composition that one function is the inverse of another. DOK 1 (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. DOK 2 (+) Produce an invertible function from a non-invertible function by restricting the domain. DOK 2

By the end of Grade 8, students will	**7.14 Know the difference between a recursive rule and an explicit expression for a function. Continued**			
NA	NA	NA	NA	(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. DOK 1
NA	7.14.1.C Combine standard function types using arithmetic operations. DOK 1	7.14.2.C Write arithmetic and geometric sequences both recursively and with an explicit formula DOK 2	7.14.3.C NA	**Stem** NA
By the end of Grade 8, students will	**7.15 Knows that manipulating the parameters of the symbolic rule will result in a predictable transformation of the graph.**			
NA	7.15.1.A Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. DOK 2	7.F.5.2.A NA	7.F.5.3.A NA	**Stem** NA

Mathematical Practices	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Recursive, Explicit, Inverse Functions, Arithmetic, Geometric, Sequence, Exponential Function, Linear Function** **See Standards for Mathematical Practices Appendix B**

Linear, Quadratic, and Exponential Models Domain	**Students will understand that:** Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will	**7.16 Describe characteristic graph, table, and equation formats for linear, exponential, and quadratic functions.**			
Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally	7.16.1.A Distinguish between situations that can be modeled with linear functions and with exponential functions. DOK 2	7.16.2.A Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. DOK 3	7.16.3.A Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. DOK 2	**Stem** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. DOK 2
	7.16.1.B Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). DOK 2	7.16.2.B Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratic ally, or (more generally) as a polynomial function. DOK 2	7.16.3.B For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. DOK 1	**Stem** Interpret the parameters in a linear or exponential function in terms of a context. DOK 2
Mathematical Practices	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Linear Function, Exponential Function, Quadratic Function, Proportional Relationship, Parameter, Logarithm, Arithmetic Sequence, Geometric Sequence, and Interval.** **See Standards for Mathematical Practices Appendix B**			

Trigonometric Functions	**Students will understand:** The connection between extending the domain of trigonometric functions using the unit circle and graphing trigonometric functions in the Cartesian coordinate system to model periodic phenomena across the extended domain. That the graph of a function is a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties (amplitude, frequency, and midline). **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will	**7.17 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle**			
NA	7.17.1.A Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. DOK 2	7.17.2.A Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. DOK 3	7.17.3.A Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* DOK 2	**Stem** (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for x, π+x, and 2π−x in terms of their values for x, where x is any real number. DOK 2
NA	**NA**	**NA**	**NA**	**STEM** (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. DOK 2

By the end of Grade 8, students will	**3.18** **Model periodic phenomena with trigonometric functions**			
NA	**NA**	**NA**	**NA**	**STEM** (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. DOK 2
NA	**NA**	**NA**	**NA**	**STEM** (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. DOK 2
NA	**NA**	**NA**	**NA**	**STEM** (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. DOK 3

Mathematical Practices	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: Periodic Function, Unit Circle, Cyclical, Periodic, Amplitude, Frequency, Midline, Sine, Cosine, Tangent, Identity **See Standards for Mathematical Practices Appendix B**

Geometry

Introduction

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

8.0 Geometry

The student will develop an understanding of geometric concepts and relationships as the basis for geometric modeling and reasoning to solve problems involving one-, two-, and three-dimensional figures.

Congruence	**Students understand that:** The concept of congruence and symmetry can be understood from the perspective of geometric transformation. Once the triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures Construction is another way to visualize and create a strategic pathway to proof **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will	**8.1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.**			
. Verify experimentally the properties of rotations, reflections, and translations:	8.1.1.A Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. DOK 2	8.1.2.A Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). DOK 2	8.1.3.A Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. DOK 2	**STEM** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. DOK 2

By the end of Grade 8, students will	**8.2.** **Know how to use both verbal and symbolic language to develop arguments related to location, transformation and congruence.**			
Verify experimentally the properties of rotations, reflections, and translations:	8.2.1.A Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. DOK 2	8.2.2.A Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. DOK 2	8.2.3.A Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. DOK 3	**Stem** NA
By the end of Grade 8, students will	**8.3.** **Know what it means to prove or disprove a conjecture.**			
Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	8.3.1.A Prove theorems about lines and angles. DOK 3 Theorems include: *vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	8.3.2.A Prove theorems about triangles. DOK 3 Theorems include: *measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	8.3.3.A Prove theorems about parallelograms. DOK 3 Theorems include: *opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals*	**Stem** NA

By the end of Grade 8, students will	8.4. Know why point, line, distance along a line and distance around a circular arc are undefined.			
NA	8.4.1.A Make formal geometric constructions with a variety of tools and methods DOK 2 (*compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line*	8.4.2.A Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. DOK 2	8.4.3.A NA	**Stem** NA
Mathematical Practice	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **ASA, SSS, SAS, Angle, Circle, Perpendicular Line, Parallel Line, Line Segment, Vertical Angle, Transversal, Alternate Interior Angles, Corresponding Angles, Bisector, Congruent**. **See Standards for Mathematical Practices Appendix B**			

Similarity, Right Triangles, and Trigonometry	**Students understand that:** Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. The definition of trigonometric ratios is not only useful in solving right triangle problems but can also be applied to general triangles. **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will	**8.5** **Know that transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence**			
Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates**.**	8.5.1.A Verify experimentally the properties of dilations given by a center and a scale factor DOK 2	84.5.2.A NA	8.5.3.A NA	**Stem** NA
	8.5.1.B Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; DOK 2 *explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*	8.5.1.B Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. DOK 3	8.5.1.B Prove theorems about triangles. DOK 3 Theorems include: *a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. DOK 3

By the end of Grade 8, students will	8.6 Know the trigonometric ratios, Sine, Cosine, and Tangent.			
NA	8.6.1.A Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. DOK 2	8.6.2.A Explain and use the relationship between the sine and cosine of complementary angles. DOK 2	8.6.3.A Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.* DOK 2	**Stem** NA
Mathematical Practices	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Dilation, Scale Factor, Pythagorean Theorem, Trigonometric Ratio, Sine, Cosine, Tangent, Complementary Angles, Similar, Congruent, Acute angle.** **See Standards for Mathematical Practices Appendix B**			

Circles	**Students will understand that:** Properties of Circles can be described by theorems that integrate algebraic and geometric understanding, modeling, and proof. Properties of Circles can be used to derive an understanding of the radian measure of an angle. **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will	**8.7** **Demonstrate that all circles are similar and how the application of proportional reasoning is used to develop the concept of radian measure.**			
NA	8.7.1.A Prove that all circles are similar. DOK 3	8.7.2.A Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. DOK 2	8.7.3.A Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. DOK 3	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. DOK 3
	NA	NA	NA	**Stem** ((+) Construct a tangent line from a point outside a given circle to the circle DOK 2
Mathematical Practices	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Radian, Inscribed, Circumscribed, Chord, Central Angle, Tangent, Arc Length, Sector.** **See Standards for Mathematical Practices Appendix B**			

Expressing Geometric Properties with Equations	**Students will understand that:** Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will:	**8.8 Know the equation of a circle**			
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	8.8.1.A Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. DOK 2	8.8.2.A Derive the equation of a parabola given a focus and directrix. DOK 2	8.8.3.A Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2). DOK 3	**Stem** (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant DOK 2

By the end of Grade 8, students will:	8.9 Demonstrate that the distance formula is an application of the Pythagorean Theorem			
Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	8.9.1.A Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). DOK 2	8.9.2.A Find the point on a directed line segment between two given points that partitions the segment in a given ratio. DOK 2	8.9.3.A Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.* DOK 2	**Stem** NA
Mathematical Practices	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Distance Formula, Focus, Directrix, Axis of Symmetry, Completing the Square, Coordinate Plane, Directed Line Segment, and Pythagorean Theorem.** See Standards for Mathematics Practice Appendix 2			

Geometric Measurement and Dimension	**Students will understand that:** Perimeter, Area, and Volume of Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of shape (composing, decomposing or slicing) correspond to algebraic changes in their equations. **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will	**8.10 Know strategies for dissection and partitioning that support the visualizations necessary to build informal arguments.**			
Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems	8.10.1.A Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. DOK 3	8.10.2.A Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.* DOK 2	8.10.3.A Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. DOK 2	**Stem** (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. DOK 3
Mathematical Practices	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Cross-section, Cylinder, Pyramid, Cone, Circumference, Area** **See Standards for Mathematical Practices Appendix 2**			

Modeling with Geometry	**Students will understand that:** Real-world situations are not organized and labeled for analysis; formulating flexible geometric models, representing such models, and analyzing them is a creative process. The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will	**8.11 Know that modeling is the process of choosing and using appropriate mathematics to analyze and understand geometric situations.**			
Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	8.11.1.A Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* DOK 2	8.11.2.A Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).* DOK 2	8.11.3.A Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).* DOK 2	**Stem** NA
Mathematical Practices	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Geometric Modeling, Approximation, Density, Design Problem** **See Standards for Mathematical Practice Appendix 2**			

DATA ANALYSIS and PROBABILITY

Introduction

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling

Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient

9.0 DATA ANALYSIS and PROBABILITY
Students will develop an understanding of Data Analysis and Probability by solving problems in which there is a need to collect, appropriately represent, and interpret data; to make inferences or predictions and to present convincing arguments; and to model mathematical situations to determine the probability.

Interpreting Categorical and Quantitative Data	**Students will understand that:** Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. When making statistical models, technology is valuable for varying assumptions, exploring consequences and comparing predictions with data. Causation implies correlation yet correlation does not imply causation. **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will	**9.1 Know that Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread.**			
Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	9.1.1.A Represent data with plots on the real number line (dot plots, histograms, and box plots). DOK 1	9.1.2.A Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. DOK 2	9.1.3.A Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). DOK 2	**Stem** NA
By the end of Grade 8, students will	**9.2 Know that the shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range).**			

NA	9.2.1.A Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. DOK 2	9.2.2.A Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. DOK 2	9.2.3.A Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. DOK 2	**Stem** NA
By the end of Grade 8, students will:	**9.3 Know strategies for fitting a function to a data display and informally assessing the fit.**			
Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	9.3.1.A Informally assess the fit of a function by plotting and analyzing residuals. DOK 2	9.3.2.A Fit a linear function for a scatter plot that suggests a linear association. DOK 2	9.3.3.A Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. DOK 2	9.3.4.A Compute (using technology) and interpret the correlation coefficient of a linear fit. DOK 2
	NA	NA	NA	9.3.4.B Distinguish between correlation and causation DOK 2

Mathematical Practices	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Dot Plot, Histogram, Box Plot, Scatter Plot, Inter-quartile Range, Standard Deviation, Measure of Center, Outliers, Normal Distribution, Skewed Distribution, Correlation Coefficient, Two-Way Frequency Table.** **See Standards for Mathematical Practices Appendix 2**

Making Inferences and Justifying Conclusions	**Students will understand that:** The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. Collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments.			
By the end of Grade 8, students will:	**9.4 Recognize the purposes of and differences among sample surveys, experiments, and observational studies.**			
Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept	9.4.1.A Understand statistics as a process for making inferences about population parameters based on a random sample from that population. DOK 2	9.4.2.A Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. DOK 3 *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	9.4.3.A Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each DOK 3	**Stem** NA

NA	9.4.1.B Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. DOK 3 .	9.4.1.B se data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant DOK 3	9.4.1.B Evaluate reports based on data. DOK 3	**Stem** NA
Mathematical Practices	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Randomization, Sample Survey, Experiment, Observational Study, Inferences, Population Parameters, Simulation, Population Mean, Population Proportion** **See Standards for Mathematical Practices Appendix 2**			

Conditional Probability and the Rules of Probability	**Students will understand that:** In a probability model, sample points represent outcomes and combine to make up events. The probabilities of the events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables **By the end of this learning progression, students will be able to…**			
By the end of Grade 8, students will	**9.5 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities**			
NA	9.5.1.A Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). DOK 2	9.5.2.A Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, ***and use this characterization to determine if they are independent.*** DOK 2	9.5.3.A Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. DOK 2	**Stem** (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. DOK 2
	NA	NA	NA	**Stem** (+) Use permutations and combinations to compute probabilities of compound events and solve problems DOK 2

By the end of Grade 8, students will:	**9.6 Recognize the concepts of conditional probability and independence in everyday language and everyday situations.**			
NA	9.6.1.A Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities DOK 2	9.6.2.A Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. DOK 2	9.6.3.A Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A,* and interpret the answer in terms of the model. DOK 2	Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model. DOK 2
By the end of Grade 8, students will:	**9.7 Calculate expected values and use them to solve problems**			
NA	NA	NA	NA	**Stem** (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. DOK 2

By the end of Grade 8, students will:	9.7 **Calculate expected values and use them to solve problems** Continued			
NA	NA		NA	**Stem** (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. DOK 2
	NA	NA	NA	**Stem** (I+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value DOK 2
	NA	NA	NA	**Stem** (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value DOK 2

By the end of Grade 8, students will:	**9.8 Use probability to evaluate outcomes of decisions**			
NA	NA		NA	**Stem** (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. DOK 2
	NA		NA	**Stem** (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). DOK 2
	NA		NA	**Stem** (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game) DOK 2
Mathematical Practices	Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Conditional Probability, Unions, Intersections, Complements, Independence, Sample Space** **See Standards For Mathematical Practices Appendix 2**			

Appendix A

Note on courses & transitions » Courses & Transitions
Adopted From Common Core

The high school portion of the Standards for Mathematical Content specifies the mathematics all students should study for college and career readiness. These standards do not mandate the sequence of high school courses. However, the organization of high school courses is a critical component to implementation of the standards. To that end, sample high school pathways for mathematics – in both a traditional course sequence (Algebra I, Geometry, and Algebra II) as well as an integrated course sequence (Mathematics 1, Mathematics 2, Mathematics 3) –have been made available by the Common Core State Standards.

The standards themselves do not dictate curriculum, pedagogy, or delivery of content. In particular, states may handle the transition to high school in different ways. For example, many students in the U.S. today take Algebra I in the 8th grade, and in some states this is a requirement. The K-7 standards contain the prerequisites to prepare students for Algebra I by 8th grade, and the standards are designed to permit states to continue existing policies concerning Algebra I in 8th grade.

A second major transition is the transition from high school to post-secondary education for college and careers. The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary **defined by (+) symbols in these standards.**

Indeed, some of the highest priority content for college and career readiness comes from Grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as Grades 6-8. It is important to note as well that cut scores or other information generated by assessment systems for college and career readiness should be developed in collaboration with representatives from higher education and workforce development programs, and should be validated by subsequent performance of students in college and the workforce.

Traditional Pathway: High school Algebra I

The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. The critical areas, called units, deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

Critical Area 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 4: In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Critical Area 5: In this unit, students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

High School Algebra 1

Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
Relationships Between Quantities and Reasoning with Equations	Linear and Exponential Relationships	Descriptive Statistics	Expressions and Equations	Quadratic Functions and Modeling
Reason quantitatively and use units to solve problems. Interpret the structure of expressions. Create equations that describe numbers or relationships. Understand solving equations as a process of reasoning and explain the reasoning. Solve equations and inequalities in one variable.	Extend the properties of exponents to rational exponents. Solve systems of equations. Represent and solve equations and inequalities graphically. Understand the concept of a function and use function notation. Interpret functions that arise in applications in terms of a context. Analyze functions using different representations. Build a function that models a relationship between two quantities. Build new functions from existing functions. Construct and compare linear, quadratic, and exponential models and solve problems. Interpret expressions for functions in terms of the situation they model.	Summarize, represent, and interpret data on a single count or measurement variable. Summarize, represent, and interpret data on two categorical and quantitative variables. Interpret linear models	Interpret the structure of expressions. Write expressions in equivalent forms to solve problems. Perform arithmetic operations on polynomials. Create equations that describe numbers or relationships. Solve equations and inequalities in one variable. Solve systems of equations.	Use properties of rational and irrational numbers. Interpret functions that arise in applications in terms of a context. *Analyze functions using different representations.* Build a function that models a relationship between two quantities. Build new functions from existing functions. Construct and compare linear, quadratic, and exponential models and solve problems.

Traditional Pathway: Geometry

The fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas, organized into six units are as follows.

Critical Area 1: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 2: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on students' work with quadratic equations done in the first course. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

Critical Area 3: Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

Critical Area 4: Building on their work with the Pythagorean theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in the first course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

Critical Area 5: In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done in the first course, to determine intersections between lines and circles or parabolas and between two circles.

Critical Area 6: Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

Geometry					
Unit 1	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**
Congruence, Proof, and Constructions	Similarity, Proof, and Trigonometry	Extending to Three Dimensions	Connecting Algebra and Geometry through Coordinates	Circles With and Without Coordinates	Applications of Probability
Experiment with transformations in the plane. Understand congruence in terms of rigid motions. Prove geometric theorems. Make geometric constructions.	Understand similarity in terms of similarity transformations. Prove theorems involving similarity. Define trigonometric ratios and solve problems involving right triangles. Apply geometric concepts in modeling situations. Apply trigonometry to general triangles.	Explain volume formulas and use them to solve problems. Visualize the relation between two-dimensional and three-dimensional objects. Apply geometric concepts in modeling situations.	Use coordinates to prove simple geometric theorems algebraically. Translate between the geometric description and the equation for a conic section.	Understand and apply theorems about circles. Find arc lengths and areas of sectors of circles. Translate between the geometric description and the equation for a conic section. Use coordinates to prove simple geometric theorem algebraically. Apply geometric concepts in modeling situations.	Understand independence and conditional probability and use them to interpret data. Use the rules of probability to compute probabilities of compound events in a uniform probability model. Use probability to evaluate outcomes of decisions.

Traditional Pathway: Algebra II

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include polynomial, rational, and radical functions.2 Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas for this course, organized into four units, are as follows:

Critical Area 1: This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Critical Area 2: Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

Critical Area 3: In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Critical Area 4: In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

Algebra 2

Unit 1	Unit 2	Unit 3	Unit 4
Polynomial, Rational, and Radical Relationships	Trigonometric Functions	Modeling with Functions	Inferences and Conclusions from Data
Perform arithmetic operations with complex numbers. Use complex numbers in polynomial identities and equations. Interpret the structure of expressions. Write expressions in equivalent forms to solve problems. Perform arithmetic operations on polynomials. Understand the relationship between zeros and factors of polynomials. Use polynomial identities to solve problems. Rewrite rational expressions. Understand solving equations as a process of reasoning and explain the reasoning. Represent and solve equations and inequalities graphically. Analyze functions using different representations.	Extend the domain of trigonometric functions using the unit circle. Model periodic phenomena with trigonometric function. Prove and apply trigonometric identites.	Create equations that describe numbers or relationships. Interpret functions that arise in applications in terms of a context. Analyze functions using different representations. Build a function that models a relationship between two quantities. Build new functions from existing functions. Construct and compare linear, quadratic, and exponential models and solve problems.	Summarize, represent, and interpret data on single count or measurement variable. Understand and evaluate random processes underlying statistical experiments. Make inferences and justify conclusions from sample surveys, experiments and observational studies. Use probability to evaluate outcomes of decisions.

Integrated Pathway: Mathematics I

The fundamental purpose of Mathematics I is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Mathematics 1 uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

Critical Area 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

Critical Area 4: This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 5: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 6: Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

MATH 1 Critical Areas (Big Ideas)					
Unit 1	**Unit 2**	**Unit 3†**	**Unit 4**	**Unit 5**	**Unit 6**
Relationships Between Quantities	Linear and Exponential Relationships	Reasoning with Equations	Descriptive Statistics	Congruence, Proof, and Constructions	Connecting Algebra and Geometry through Coordinates
Reason quantitatively and use units to solve problems. Interpret the structure of expressions. Create equations that describe numbers or relationships.	Represent and solve equations and inequalities graphically. Understand the concept of a function and use function notation. Interpret functions that arise in applications in terms of a context. Analyze functions using different representations. Build a function that models a relationship between two quantities. Build new functions from existing functions. Construct and compare linear, quadratic, and exponential models and solve problems. Interpret expressions for functions in terms of the situation they model.	Understand solving equations as a process of reasoning and explain the reasoning. Solve equations and inequalities in one variable. Solve systems of equations.	Summarize, represent, and interpret data on a single count or measurement variable. Summarize, represent, and interpret data on two categorical and quantitative variables. nterpret linear models.	Experiment with transformations in the plane. Understand congruence in terms of rigid motions. Make geometric constructions.	Use coordinates to prove simple geometric theorems algebraically.

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.
†Note that solving equations and systems of equations follows a study of functions in this course. To examine equations before functions, this unit could be merged with Unit 1.

Integrated Pathway: Mathematics II

The focus of Mathematics II is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Mathematics I as organized into 6 critical areas, or units. The need for extending the set of rational numbers arises and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, round out the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. In Unit 3, students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows $x+1 = 0$ to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

Critical Area 2: Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Critical Area 3: Students begin this unit by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

Critical Area 4: Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

Critical Area 5: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

Critical Area 6: In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

MATH 2 Critical Areas

UNIT 1	UNIT 2	UNIT 3	UNIT 4	UNIT 5	UNIT 6
Extending the Number System	Quadratic Functions and Modeling	*Expressions and Equations	Applications of Probability	Similarity, Right Triangle Trigonometry, and Proof	Circles With and Without Coordinates
Extend the properties of exponents to rational exponents. Use properties of rational and irrational numbers. Perform arithmetic operations with complex numbers. Perform arithmetic operations on polynomials.	Interpret functions that arise in applications in terms of a context. Analyze functions using different representations. Build a function that models a relationship between two quantities. Build new functions from existing functions. Construct and compare linear, quadratic, and exponential models and solve problems.	Interpret the structure of expressions. Write expressions in equivalent forms to solve problems. Create equations that describe numbers or relationships. Solve equations and inequalities in one variable. Use complex numbers in polynomial identities and equations. Solve systems of equations	Understand independence and conditional probability and use them to interpret data. Use the rules of probability to compute probabilities of compound events in a uniform probability model. Use probability to evaluate outcomes of decisions.	Understand similarity in terms of similarity transformations. Prove geometric theorems. Prove theorems involving similarity. Use coordinates to prove simple geometric theorems algebraically. Define trigonometric ratios and solve problems involving right triangles. Prove and apply trigonometric identities	Understand and apply theorems about circles. Find arc lengths and areas of sectors of circles. Translate between the geometric description and the equation for a conic section. Use coordinates to prove simple geometric theorem algebraically. Explain volume formulas and use them to solve problems.

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.
†Note that solving equations follows a study of functions in this course. To examine equations before functions, this unit could come before Unit 2.

Integrated Pathway: Mathematics III

It is in Mathematics III that students pull together and apply the accumulation of learning that they have from their previous courses, with content grouped into four critical areas, organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational, and radical functions.3 They expand their study of right triangle trigonometry to include general triangles. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

Critical Area 2: This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Critical Area 3: Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles open up the idea of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

Critical Area 4: In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

MATH 111 Critical Areas (Big Ideas)

Unit 1	Unit 2	Unit 3	Unit 4
Inferences and Conclusions from Data	Polynomial, Rational, and Radical Relationships	Trigonometry of General Triangles and Trigonometric Functions	Mathematical Modeling
Summarize, represent, and interpret data on single count or measurement variable. Understand and evaluate random processes underlying statistical experiments. Make inferences and justify conclusions from sample surveys, experiments, and observational studies. Use probability to evaluate outcomes of decisions	Use complex numbers in polynomial identities and equations. Interpret the structure of expressions. Write expressions in equivalent forms to solve problems. Perform arithmetic operations on polynomials. Understand the relationship between zeros and factors of polynomials. Use polynomial identities to solve problems. Rewrite rational expressions. Understand solving equations as a process of reasoning and explain the reasoning. Represent and solve equations and inequalities graphically. Analyze functions using different representations.	Apply trigonometry to general triangles. Extend the domain of trigonometric functions using the unit circle. Model periodic phenomena with trigonometric function	Create equations that describe numbers or relationships. Interpret functions that arise in applications in terms of a context. Analyze functions using different representations. Build a function that models a relationship between two quantities. Build new functions from existing functions. Construct and compare linear, quadratic, and exponential models and solve problems. Visualize relationships between two-dimensional and three-dimensional objects. Apply geometric concepts in modeling situations.

Appendix B
Standards for Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut.

In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Appendix C
Glossary
Adopted from Common Core

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Associative property of addition. $(a + b)+ c = a + (b + c)$

Associative property of multiplication. $(a \times b) \times c = a \times (b \times c)$

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team. Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. $a \times b= b \times a$ (multiplication) $a + b= b + a$ (addition)

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See*: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. a + 0 = 0+ a= a

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form a or –a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations.

The properties of operations. Herea, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition (a + b) + c = a + (b + c)
Commutative property of addition a + b = b + a
Additive identity property of 0 a + 0 = 0 + a = a
Existence of additive inverses For every a there exists –a so that a + (–a) = (–a) + a= 0.
Associative property of multiplication (a × b)× c = a × (b × c)
Commutative property of multiplication a × b = b × a
Multiplicative identity property of 1 a ⊕ 1= 1
Existence of multiplicative inverses For every a ≠ 0 there exists 1/a so that a 1/a= 1/a × a = 1.
Distributive property of multiplication over addition a ×(b + c) = a × b + a × c

Properties of equality.

The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality a = a
Symmetric property of equality If a = b and b = c, then a = c.
Transitive property of equality If a = b and b = c, then a = c.
Addition property of equality If a = b, then a + c = b + c.
Subtraction property of equality If a = b, then a – c = b – c.
Multiplication property of equality If a = b, then a × c = b × c.
Division property of equality If a = b and c ≠ 0, then a ÷ c = b ÷ c.
Substitution property of equality If a=b, then b may be substituted for a in any expression containing a.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or – a/b for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3,....

[1]Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/ standards/mathglos.html, accessed March 2, 2010.

[2]Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[3]Adapted from Wisconsin Department of Public Instruction, op. cit.

[4]To be more precise, this defines the arithmetic mean.

[5]Adapted from Wisconsin Department of Public Instruction, op. cit.

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